Human Capital Investment and Preference Formation

Lin Zhang†

Abstract

This paper develops a model of parental investment in her child’s education that incorporates parental altruism with time-preference formation and human capital formation, and assumes decreasing marginal impatience. It explains the behavior which is consistent with empirical evidence but cannot be predicted by previous studies: A parent who invests more in her child’s educational as her income decreases. The necessary and sufficient condition is provided for this behavior. This model also predicts that the child can consume less as income increases and that the parent can sacrifice her own consumption to increase educational investment. Logarithmic utility functions can exhibit these properties.

Keywords: Time Preference Formation; Human Capital Formation; Educational Investment.

JEL Classification: D9; I2; J24.

1. Introduction

Education develops a child’s human capital, but requires the deployment of parental resources. In a standard parental-altruism model with an exogenous time preference, a parent’s investment in her child’s education increases with her income level. Intuitively, the rich spend more on their children’s education. However, several empirical studies (Bennett and Xie, 2003; Charles, Roscigno, and Torres, 2007; and Mangino, 2012) find that in the US, families with a high annual income have low college transition rates and that African-Americans, Hispanics, and Asians are more likely to go to college than white people. The standard parental-altruism model with an exogenous time preference cannot predict this counter-intuitive phenomenon. The aim of this paper is to show that a parent can increase educational investment as her income decreases in a parental-altruism model with endogenous child time preference.

This paper employs the Bhatt and Ogaki’s (2012) tough love model, in which the child’s discount factor could be influenced by the parent. A number of empirical studies show that the children’s time preferences are related to

† Email address: z-lin@seiryo-u.ac.jp
their parents’ behavior (Delaney and Doyle, 2012; Dohmen et al., 2006). In contrast to the Bhatt and Ogaki’s (2012) assumption that the child’s discount factor decreases with parental transfers, this paper assumes that the child’s discount factor increases with the parent’s investment in the child’s education. Intuitively, a child who receives more education becomes more patient. This assumption implies decreasing marginal impatience (DMI) for educational investment, which is in line with the models of Becker and Mulligan (1997) and Stern (2006), who claim that people must expend current resources to increase their appreciation of the future. This paper is also consistent with the theoretical analysis of Doepke and Zilibotti (2008), in which parents invest resources to shape their children’s preferences in a desired way. The assumption of decreasing marginal impatience is supported by many empirical evidences, for example, Ikeda, Ohtake, and Tsutsui (2006), Lawrance (1991), Samwick (1998). In particular, a number of empirical studies that describe a series of field experiments in different countries and areas show that individuals with higher education levels are relatively more patient (Bauer and Chytilová, 2009; Eisenhauer and Ventura, 2006; Burks et al., forthcoming; Harrison, Lau, and Williams, 2002; Reimers et al., 2009).

This paper employs a parental-altruism model with overlapping generations. The parent (the mother) divides her income between consumption and investment in her child’s education (a son). Education not only develops the child’s human capital but also causes him to become more patient. Hence, this setup is different from that of Bhatt and Ogaki (2012) in that this paper includes the formation of not only time preference but also human capital. In other words, the parent determines not only the child’s income allocation between the two periods but also the level of his level. This paper is also related to Doepke and Zilibotti (2008), in which parents exert costly efforts to shape their children’s preference. However, in this paper the parent invests a part of her income to develop the child’s human capital and instill patience.

In describing the behavior of the parent and the child, some definitions will be useful. The child is considered frugal when his first-period consumption decreases with income. Namely, the frugal child consumes less even if his income is raised. The parent is regarded as self-sacrificing when she decreases her own consumption levels to invest more in her child’s education. Neither of these behaviors can be predicted by the standard parental-altruism model with exogenous time preference. Furthermore, this paper shows that a decrease in the parent’s endowment income can lead to more investment in education, which is consistent with empirical studies.

The principal findings of this paper are follows. First, the child is frugal if and only if his marginal patience in forming human capital is sufficiently larger than his absolute risk-aversion. Second, the parent decides to be self-sacrificing if and only if the child’s utility evaluated by the parent is convex in educational investment. Third, the study provides the necessary and sufficient condition under which a decrease in the parent’s endowment income leads to more investment in education. The parent’s self-sacrificing behavior is a necessary condition for educational investment to be decreasing with endowment income. Fourth, an example of a logarithmic utility function allowing these conditions is shown.

The original contributions of this paper are as follows. It explains behaviors that cannot be predicted by the standard model but that are supported by empirical evidences and provides the necessary and sufficient conditions.
for these behaviors. A functional-form example shows the specific terms of these conditions. Also, this paper introduces the formation of time preference and human capital, and relates them to education. The assumption of decreasing marginal impatience in this model is widely supported by recent empirical studies.

The rest of this paper is organized as follows. Section 2 introduces the theoretical model. Section 3 describes the behaviors of the parent and the child. Section 4 presents an example and discusses the findings. Section 5 concludes the paper.

2. The model

Consider a family consisting of a parent (a mother) and a child (a son) both of whom live for two periods. The parent gives birth to a child in her second period, which therefore overlaps with the child’s first period. In the first period, the parent allocates her endowment income $y_1$ among first-period consumption $c_1$, investment in the child’s education $e$, and savings $s_1$. In her second period, the parent consumes the savings from period 1:

$$d_1 = Rs_1 = R(y_1 - c_1 - e),$$  \hspace{1cm} (1)

where $R$ is the gross interest rate, and $d_1$ the parent’s second-period consumption.

The parent is altruistic toward her child, so her life-time utility reads

$$V = (1 - \gamma)[v(c_1) + \beta v(d_1)] + \gamma \beta W$$
$$= (1 - \gamma)[v(c_1) + \beta v(d_1)] + \gamma \beta[u(c_2) + \beta u(d_2)],$$  \hspace{1cm} (2)

where $\beta > 0$ is a constant discount factor of the parent for herself, $\bar{\beta} > 0$ a constant discount factor of the parent for the child, and $\gamma \in [0, 1]$ the degree of parental altruism. Both utility functions $u(.)$ and $v(.)$ are concave. Note that

$$W = u(c_2) + \beta u(d_2)$$  \hspace{1cm} (3)

is the altruistic component and it represents the child’s maximized utility evaluated by the parent.

In the child’s first period, he supplies human capital $h$ and receives wage income at the rate of $w$. The human capital of the child $h$ depends on the parent’s expenditure on education $e$ in the form

$$h = h(e).$$  \hspace{1cm} (4)

I stipulate the following assumption about human capital formation.
Assumption 1 The function of human capital $h(e)$ is increasing and concave in educational investment $e$.

In this context, the parent can control her child’s income by choosing the amount of education expense. The child’s budget constraint reads

$$d_2 = R s_2 = R (wh(e) - c_2),$$

(5)

where $c_2$ denotes the child’s first-period consumption, $d_2$ the second-period consumption, and $s_2$ the child’s saving.

The child’s preference is

$$U = u(c_2) + \beta u(d_2),$$

(6)

where $\beta > 0$ denotes the discount factor of the child, which measures the degree of the child’s patience and is exogenous for the child when he makes inter-temporal decisions. However, the parent can influence the discount factor $\beta$ (e.g., Delaney and Doyle, 2012; Dohmen et al., 2006). To incorporate this intuitive notion, I assume that the child’s discount factor $\beta$ is dependent on the parent’s education expense:

$$\beta = \beta(e).$$

(7)

Specifically, assume that the child’s discount factor $\beta$ increases with his human capital, which in turn increases with the parent’s expense on education. Therefore, the child’s discount factor $\beta$ is an increasing function of the parent’s education expenditure $e$.

Assumption 2 The function describing the child’s discount factor, $\beta(e)$, is increasing in the parent’s educational investment $e$ ($\beta'(e) > 0$).

In other words, more investments on education causes the child to become more patient. This assumption agrees with the intuitive notion that the rate of time preference is a decreasing function of human capital and that the parent invests to shape their children’s appreciation of the future (Becker and Mulligan, 1997; Stern 2006; Doepke and Zilibotti 2008).

As mentioned above, an increase in the parent’s educational investment has two impacts on the child: increases in income and in patience. The altruistic parent chooses the level of educational investment to control the child’s consumption level. The next section employs the following terminologies to characterize the behavior of the parent and child.
**Definition 1** The child is considered *frugal* when his first-period consumption decreases with an increase in income.

**Definition 2** The parent is regarded as *self-sacrificing* when she decides to decrease her own consumption levels to invest more in her child’s education.

### 3. Endogenous discount factor and educational investment

#### 3.1 The behavior of the parent and the child

Because parental altruism is assumed in this model, the parent incorporates the child’s utility in her own utility-maximizing problem. The parent anticipates her child’s behavior and makes corresponding decisions on consumption and education expense. She can control the child’s income and discount factor by adjusting the education expense. Therefore, we solve the child’s utility-maximizing problem first.

The child maximizes his life-time utility (6) subject to the budget constraint (5), taking $\beta(e)$, $h(e)$, $R$, and $w$ as given. The first-order condition of the child is

$$u'(c_2) = \beta(e)Ru'(d_2). \tag{8}$$

The inter-temporal budget constraint (5) and first-order condition (8) jointly determine the child’s optimal consumption levels $c_2(\beta(e), h(e), R, w)$ and $d_2(\beta(e), h(e), R, w)$. Thus, the child’s consumption $c_2$ and $d_2$ are functions of the educational investment $e$, which is controlled by the parent.

The parent maximizes her life-time utility (2) subject to the budget constraint (1), and the first-order conditions are

$$v'(c_1) = \tilde{\beta}Rv'(d_1) \tag{9}$$

and

$$\left(\frac{1-\gamma}{\gamma}\right)Rv'(d_1) = u'(c_2) \frac{d(c_2)}{de} + \tilde{\beta}u'(d_2) \frac{d(d_2)}{de}, \tag{10}$$

which describe how the parent decides to allocate her income between consumption and educational investment. The inter-temporal budget constraint (1) and first-order conditions (9) and (10) jointly determine the parent’s optimal consumption levels $c_1(y, \tilde{\beta}, \bar{\beta}, \gamma, R, w)$ and $d_1(y, \tilde{\beta}, \bar{\beta}, \gamma, R, w)$, and optimal expenditure on the child’s education $e(y, \tilde{\beta}, \bar{\beta}, \gamma, R, w)$.

#### 3.2 Effects of educational investment

The parent’s educational investment affects the child’s consumption through two channels: the child’s human
capital and discount factor. Higher educational investment increases the child’s income, the source of his consumption, raising his patience level and leading to a lower weightage for first-period consumption. In other words, the parent controls not only the child’s degree of patience but also his income level. In this model, higher income does not necessarily lead to greater first-period consumption, because the child becomes more patient. Proposition 1 demonstrates the necessary and sufficient condition under which the child consumes less in the first period even though his income increases.

**Proposition 1** When the parent increases educational investment, the child is frugal if and only if

$$\frac{\beta'(e)}{h'(e)} > -\frac{u''(d_2)}{u'(d_2)} \beta(e) R_w.$$

The proof of Proposition 1 is provided in the Appendix. Notice that \(\frac{\beta'(e)}{h'(e)} = \frac{d(\beta)}{d(h)}\), which represents the marginal effect of human capital accumulation on the degree of patience. Furthermore, \(-\frac{u''(d_2)}{u'(d_2)}\) is the measure of absolute risk aversion in period 2. When the marginal effect on patience of accumulation of human capital is sufficiently larger than the absolute risk aversion, human capital can raise the level of patience to a sufficient degree for the child to become more patient and give a higher weight to second-period consumption. Therefore, he gives up part of current consumption in period 1 in favor of future consumption even if his total income increases. At the same time, the child’s second-period consumption always increases with educational investment whether he is frugal or not.

The parent allocates her endowment income between consumption and educational investment. Her income allocation depends on how much utility the educational investment generates.

**Proposition 2** The parent is self-sacrificing if and only if the child’s utility as evaluated by the parent, \(W\), is convex in educational investment \(e\).

The proof of Proposition 2 is provided in the Appendix. Proposition 2 provides the necessary and sufficient condition that leads the parent to decrease her own consumption in order to increase educational investment. The convexity of \(W\) in educational investment \(e\) shows how the parent’s utility depends on her expenditures on the child’s education. Under the condition in Proposition 2, the parent sacrifices her own consumption because her investment in the child’s education brings her higher utility.

### 3.3 Parental income and educational investment

The previous sub-section discusses the relationship between consumption and educational investment. This sub-section investigates how the parent’s exogenous endowment income affects her educational investment decision.
Recall from the previous sections that the parent’s educational investment $e (y_1, \bar{\beta}, \bar{y}, R, w)$ depends on her income $y_1$, which is exogenously given. With a lower income, the parent can choose to decrease education expense to maintain her own consumption. However, in this model, with an endogenous discount factor, she can also choose to invest more in her child’s education to achieve a higher utility. Proposition 3 states the necessary and sufficient condition under which the parent increases educational investment even though her income decreases.

**Proposition 3** The parent’s educational investment decreases with endowment income if and only if

$$ \frac{d^2 W}{de^2} > -\frac{R^2(1-\gamma)}{\gamma} \frac{u''(c_1)u''(d_1)}{u''(c_1) + \bar{\beta}Ru''(d_1)}. $$

Notice that $-\frac{R^2(1-\gamma)}{\gamma} \frac{u''(c_1)u''(d_1)}{u''(c_1) + \bar{\beta}Ru''(d_1)}$ is positive. Recall from Proposition 2 that the parent is self-sacrificing when $\phi > 0$. Therefore, the parent being self-sacrificing is a necessary condition for educational investment to be decreasing with endowment income. When $\frac{d^2 W}{de^2}$ is positive and sufficiently large, educational investment can generate so much utility that the parent chooses to invest more in education even though her income is lower.

### 4. Discussions

#### 4.1 An example of logarithmic utility

The previous section provides the necessary and sufficient conditions under which the child is frugal and the parent is self-sacrificing, investing more in education when her income decreases. This subsection employs a specific functional form to show the existence of these behaviors.

Assume that the parent and the child each have logarithmic utility:

$$ v(x) = u(x) = \log(x). \quad (11) $$

This functional form meets the usual assumption about the utility functions and yields analytical results.

**Corollary 1** Under logarithmic utility, when the parent increases educational investment, the child is frugal if and only if

$$ \frac{\beta'(e)}{h'(e)} > \frac{1 + \beta(e)}{h(e)}. $$

Consistent with Proposition 1, the child gives up some current consumption in period 1 and saves more with...
increasing income when the marginal effect on patience of human capital accumulation is sufficiently large.

**Corollary 2** Under logarithmic utility, the parent is self-sacrificing if and only if \( M' > 0 \), where

\[
M' = \frac{\beta'(e)\beta(e)(1+\beta(e))}{\beta(e)^2(1+\beta(e))^2} - \frac{1}{(1+\beta)} \left( \frac{h'(e)^2 - h'(e)h(e)}{h(e)^2} \right) + \frac{(\beta'(e))^2((\beta(e))^2 - \beta(1+2\beta(e)))}{(\beta(e)^2(1+\beta(e))^2} - (1+\beta'(e))h'(e)h(e)
\]

**Corollary 3** Under logarithmic utility, the parent’s educational investment decreases with endowment income if and only if \( M' > \frac{\gamma M^2}{1-\gamma(1+(\beta R)^{-1})} \),

where \( M = (1+\beta) \frac{h'(e)}{h(e)} + \frac{(\beta - \beta(e))\beta'(e)}{\beta(e)(1+\beta(e))} \) and \( M' = \frac{d(M)}{d(e)} \).

Consistent with Propositions 2 and 3, Corollary 2 provides a necessary condition for Corollary 3.

The proofs of Corollary 1, 2, and 3 are similar to the proofs of Propositions 1, 2 and 3. Substituting the utility functions leads to the conditions in these corollaries. The logarithmic utility function form thus allows the child to be frugal and the parent to be self-sacrificing and invest more in education when the endowment income decreases.

Notice that these corollaries under the assumption of these specific utility functions involve the properties of human capital \( h(e) \) and the child’s discount factor \( \beta(e) \), but they do not require these specific functional forms. This paper gives the basic assumptions that describe how educational investment is related to human capital \( h(e) > 0, h'(e) > 0 \) and \( h''(e) < 0 \) and the child’s discount factor \( \beta(e) > 0 \) and \( \beta'(e) > 0 \) and can satisfy the necessary and sufficient conditions discussed in this paper. Further assumptions about the form of human capital \( h(e) \) and the child’s discount factor \( \beta(e) \) are not essential and they are beyond the scope of this paper.

### 4.2 Why endogenous discount factor?

This paper investigates the consumption behavior of parent and child and educational investment by the parent in a parental-altruism model where the child’s discount factor and income are determined by the parent’s educational investment. It finds that the child can be frugal, and that the parent can be self-sacrificing and invest more in education even though her endowment income decreases. These behaviors depend on the marginal effect on patience of human capital accumulation and on the properties of the child’s utility as evaluated by the parent. However, a standard parental-altruism model where the child’s discount factor is exogenously given cannot predict these behaviors.

In the standard model with an exogenous discount factor, the child’s allocation of income between two periods is exogenous because the degree of patience is constant. The consumption level in each period is increasing with income. Hence, an increase in the child’s income definitely leads to an increase in his first-period consumption. It is impossible for the child to consume less with increasing income.

With the exogenous discount factor of the child, greater educational investment increases only the child’s
income. The child’s utility function as evaluated by the parent is assumed to be concave in educational investment. In other words, educational investment cannot raise the child’s utility to the altruistic parent sufficiently for the latter to invest more in the child’s education, sacrificing her own consumption, or to do so when her endowment income decreases.

Therefore, the standard parental-altruism model with an exogenous discount factor is not consonant with the child being frugal or the parent self-sacrificing. However, when the child’s discount factor increases with the parent’s educational investment, the parent can cause her child to be more patient and becomes willing to forego her own consumption to invest in the child’s education. Thus the endogenous child discount factor as introduced in this paper can explain these behaviors.

5. Concluding Remarks

This paper develops a parental-altruism model where the child’s endogenous discount factor and human capital are increasing with the parent’s educational investment. In this setting, the child can consume less in the first period when his income increases. This depends on the marginal effect on patience of accumulation of human capital. The parent is willing to forego her own consumption to invest in the child’s education, and even invests more in education when her endowment income is decreased. This depends on how educational investment enhances the child’s utility as evaluated by the parent. The logarithmic utility function allows these behaviors.

This paper also shows that the standard parental-altruism model with an exogenous discount factor cannot predict these behaviors.

Appendix

A. Proof of Proposition 1

**Proposition 1** When the parent increases educational investment, the child is frugal if and only if

\[
\frac{\beta'(e)}{h'(e)} > -\frac{u''(d_1)}{u'(d_1)} \beta(e) R w.
\]

**Proof**

For a child, the educational investment is exogenous. How he allocates the income is based on

\[
u'(e^c_2) = \beta(e) R u'(d_2),
\]

(A.1)
and he faces the budget constraint

\[ d_2 + R c_2 = R w(e). \]  \hfill (A.2)

Taking the derivatives of (A.1) and (A.2) with respect to educational investment \( e \) leads to

\[ u''(c_2) \frac{d(c_2)}{d(e)} = R \beta'(e) u'(d_2) + R \beta(e) u''(d_2) \frac{d(d_2)}{d(e)}, \]  \hfill (A.3)

\[ \frac{d(d_2)}{d(e)} + R \frac{d(c_2)}{d(e)} = R w'(e). \]  \hfill (A.4)

Combining (A.3) and (A.4) leads to

\[ \frac{d(c_2)}{d(e)} = \frac{R \beta'(e) u'(d_2) + R^2 w \beta(e) h'(e) u''(d_2)}{u''(c_2) + R^2 \beta(e) u''(d_2)}, \]  \hfill (A.5)

\[ \frac{d(d_2)}{d(e)} = \frac{R w'(e) u''(c_2) - R^2 \beta'(e) u'(d_2)}{u''(c_2) + R^2 \beta(e) u''(d_2)}. \]  \hfill (A.6)

Because \( \beta'(e) > 0, \ h'(e) > 0, \ u''(c_2) < 0, \) and \( u''(d_2) < 0, \) \( \frac{d(c_2)}{d(e)} < 0 \) and if only if \( R \beta'(e) u'(d_2) + R^2 w \beta(e) h'(e) u''(d_2) > 0, \) which implies \( \frac{\beta'(e)}{h'(e)} > -\frac{u''(d_2)}{u'(d_2)} \beta(e) R w. \) Meanwhile, \( \frac{d(d_2)}{d(e)} \) is definitely positive.

B. Proof of Proposition 2

**Proposition 2** The parent is self-sacrificing if and only if the child’s utility as evaluated by the parent, \( W, \) is convex in educational investment \( e. \)

**Proof**

The parent’s decision on how to allocate income between consumption and education expense depends on the first-order conditions

\[ v'(c_1) = \tilde{\beta} R v'(d_1) \]  \hfill (A.7)

\[ \frac{1 - \gamma}{\gamma} R v'(d_1) = u'(c_2) \frac{d(c_2)}{d(e)} + \tilde{\beta} u'(d_2) \frac{d(d_2)}{d(e)}. \]  \hfill (A.8)
where \( u'(c_2) \frac{d(c_2)}{de} + \bar{\beta}u'(d_2) \frac{d(d_2)}{de} = \frac{dW}{de} \). Because \( v'(d_1) > 0, \frac{dW}{de} > 0 \).

I differentiate the first-order condition (A.8) with respect to educational investment \( e \) in order to investigate the parent’s decision about the income allocation decision:

\[
\left(1 - \frac{\gamma}{\gamma}\right)RV''(d_1) \frac{d(d_1)}{de} \quad = \quad u''(c_2) \left( \frac{d(c_2)}{de} \right)^2 + u'(c_2) \frac{d^2(c_2)}{de^2} + \bar{\beta} [u''(d_2) \left( \frac{d(d_2)}{de} \right)^2 + u'(d_2) \frac{d^2(d_2)}{de^2}] \quad - \quad \frac{d^2W}{de^2},
\]

where \( u''(c_2) \frac{d^2(c_2)}{de^2} + u'(c_2) \frac{d^2(c_2)}{de^2} + \bar{\beta} [u''(d_2) \left( \frac{d(d_2)}{de} \right)^2 + u'(d_2) \frac{d^2(d_2)}{de^2} - \frac{d^2W}{de^2}].

Because \( v''(d_1) < 0, \frac{d(d_1)}{de} < 0 \) implies that \( \frac{d^2W}{de^2} > 0 \). Besides, we have \( \frac{dW}{de} > 0 \). Therefore, \( \frac{d(d_1)}{de} < 0 \) if and only if \( W \) is convex.

The first-order condition (A.7) implies that \( \frac{d(c_1)}{de} = \bar{\beta}R \frac{v''(d_1)}{v''(c_1)} \frac{d(d_1)}{de} \). Because \( \bar{\beta}R \frac{v''(d_1)}{v''(c_1)} > 0 \), \( \frac{d(c_1)}{de} < 0 \) if and only if \( W \) is convex.

The conditions \( \frac{d(d_1)}{de} < 0 \) and \( \frac{d(c_1)}{de} < 0 \) imply that the parent decides to sacrifices her own consumption in order to invest more in the child’s education.

C. Proof of Proposition 3

**Proposition 3** The parent’s educational investment deceases with endowment income if and only if

\[
\frac{d^2W}{de^2} > - \frac{R^2 (1 - \gamma)}{\gamma} \frac{u''(c_1)u''(d_1)}{u''(c_1) + \bar{\beta}Ru''(d_1)}.\]

**Proof**

The parent’s budget constraint reads

\[
d_1 + Rc_1 + Re = Ry_1. \quad \text{(A.10)}
\]

Differentiating (A.10) with respect to endowment income \( y_i \) leads to

\[
\left( \frac{d(d_1)}{de} + R \frac{d(c_1)}{de} + R \frac{d(e)}{dy_i} \right) = R. \quad \text{(A.11)}
\]
Because \( \frac{d(c_t)}{de} = \frac{-\beta R}{v'(c_t)} \frac{d(d_t)}{dy_1} \), \( \frac{d(e)}{dy_1} < 0 \) implies that \( (1 + \frac{\beta R}{v'(c_t)}) \frac{d(d_t)}{de} < -R \). (A.9) implies that

\[
\frac{d(d_t)}{de} = \frac{d^2W}{de^2} \frac{1}{(1-\gamma)Rv'(d_t)}.
\] (A.12)

Therefore, \( \frac{d(e)}{dy_1} < 0 \) if and only if

\[
\frac{d^2W}{de^2} > \frac{R^2(1-\gamma)}{\gamma u''(c_t)u''(d_t)} - \frac{u''(c_t)u''(d_t)}{\gamma u''(c_t) + \beta R u''(d_t)}.
\]

References


[School of Economics, Kanazawa Seiryo University]