

# Funded Pension Scheme, Endogenous Time Preference and Capital Accumulation

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## Abstract

This paper investigates the effect of the funded pension scheme on capital accumulation in general equilibrium when the time discount factor is endogenously determined. A two-period overlapping generations model with endogenous labor supply and a balanced funded pension scheme is employed. In the benchmark model with exogenous discount factor, the funded pension scheme is neutral to capital accumulation. Contrastingly, in the model with endogenous discount factor, I assume that discount factor is increasing in economy-wide average saving level, and find that the funded pension scheme can be negative to capital accumulation.

**Keywords:** Funded Pension Scheme; Endogenous Discount Factor; Capital Accumulation; Overlapping Generations Model.

**JEL Classification:** D9, E22, H55

## 1. Introduction

The funded pension scheme as a mandatory saving instrument is shown to be completely neutral or positive to capital accumulation by previous studies, for example, de la Croix and Michel (2002), Zhang (1995) and Kaganovich and Zilcha (2012). However, some empirical studies find that funded pension system has a negative impact on capital accumulation (for example, Singh (1996)). This empirical phenomenon has been theoretically explained by several factors, for example, family altruism (Kunze (2012)). This paper finds another theoretical explanation for the negative impact from the aspect of time preference. The aim of this paper is to show that when time preference is endogenously determined, a balanced funded pension scheme is negative to capital accumulation, in contrast to the neutral effect when time preference is exogenous.

A number of theoretical studies incorporate endogenous time preference. The assumptions on endogenous

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time preference mainly include two types: with internalization and without internalization. Uzawa (1968) and Stern (2006) use individual variables as the determinants of time preference parameter and hence the individual variables are internalized. Schmitt-Grohé and Uribe (2003), Bian and Meng (2004), and Sodini (2011) assume that time preference parameter depends on economy-wide average variables which are taken as given. This paper is accordant with the latter, and assumes that discount factor is endogenously determined by economy-wide average saving level. As Schmitt-Grohé and Uribe (2003) point out, the specification without internalization makes the model computationally simpler and does not predict significantly different result from the one with internalization. This paper follows Becker and Mulligan (1997) and Stern (2006) by assuming that time preference depends on the variables which push out current consumption. Furthermore, the assumption of decreasing marginal impatience (DMI) is consistent with many empirical studies (Lawrance (1991), Samwick (1998) and Ikeda et al. (2006)).

This paper employs a two-period overlapping generations model with endogenous labor supply and a balanced funded pension scheme. General equilibrium is derived in goods, labor and capital market. The model with endogenous discount factor is compared with the benchmark model where discount factor is exogenously given. In the presence of endogenous discount factor, which is increasing in economy-wide average saving level, a balanced funded pension scheme is negative to capital accumulation. This is contrasted with the result of the benchmark model.

The remainder is organized as follows. Section 2 discusses the benchmark model where discount factor is exogenously given. Section 3 investigates the effect of funded pension scheme on capital accumulation when discount is endogenous. Section 4 concludes.

## 2. The Benchmark Model with Exogenous Time Preference

### 2.1 The Benchmark Model

Consider an economy populated by overlapping generations living for two periods (the youth and the old). I assume that time is discrete and infinite. In every period, there live two generations. In this economy, I assume constant population and that the population of each generation is one.

For generation  $t$  ( $t = 1, 2, \dots, \infty$ ) who is born in period  $t$ , she determines labor supply ( $l_t \in (0, 1)$ ) in her first period of life. The consumption levels of generation  $t$  in her first and second periods are  $c_t$  and  $d_t$  respectively.  $s_t$  denotes the saving level in her young period. When generation  $t$  is working, she receives wage at a rate of  $w_t$ .

Assume that the government imposes a funded pension scheme in the young period and gives a pension benefit in old age. The proportional pension contribution rate is  $\tau \in [0, 1]$ , and the pension benefit  $T_t$ . The budget constraints of the consumer include

$$c_t + s_t = w_t l_t (1 - \tau), \quad (1)$$

$$\text{and } d_t = R_{t+1}s_t + T_t, \quad (2)$$

where  $R_{t+1}$  denotes the interest factor in period  $t+1$ .

A balanced funded pension scheme reads

$$T_t = R_{t+1}\tau w_t l_t, \quad (3)$$

For preference, I assume logarithmic utility function as follows:

$$U_t = \ln(c_t) + \ln(1-l_t) + \beta \ln(d_t), \quad (4)$$

where  $\beta \in [0,1]$  denotes the discount factor which is exogenously given in the benchmark model. According to Stern (2006), discount factor  $\beta$  measures the consumer's appreciation of future utility. A smaller  $\beta$  implies stronger impatience.

In this economy, there is a firm which produces homogeneous goods with capital and labor. I assume that the firm has the following Cobb-Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad (5)$$

where  $K_t$  denotes the total capital in period  $t$ ,  $L_t$  the total labor demand,  $A > 0$  the total factor productivity, and  $\alpha \in (0,1)$  the output elasticity of capital.

## 2.2 The Benchmark Solution

Consumer's budget constraints (1) and (2), and preference (4) jointly determine the optimal consumption level and labor supply in young period, and the first order conditions are

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{d_t}, \quad (6)$$

$$\frac{1}{1-l_t} = \frac{\beta R_{t+1} w_t}{d_t}. \quad (7)$$

Combining equations (6) and (7) gives

$$\frac{1-l_t}{c_t} = \frac{1}{w_t}. \quad (8)$$

The equilibrium conditions of capital and labor markets read

$$K_{t+1} = s_t + \tau w_t l_t, \quad (9)$$

$$L_t = l_t. \quad (10)$$

The firm maximizes its profit and the first order conditions are

$$R_t = A \alpha k_t^{\alpha-1}, \quad (11)$$

$$w_t = A(1-\alpha)k_t^\alpha, \quad (12)$$

where  $k_t = K_t / L_t$  denotes the capital intensity. Therefore, production function (5) is also in the form

$$Y_t = A k_t^\alpha l_t. \quad (13)$$

Equation (9) and (10) imply

$$Y_t = c_t + s_t + d_{t-1} + \tau w_t l_t. \quad (14)$$

Substituting (1), (12) and (13) into (14) leads to

$$d_{t-1} = A \alpha k_t^\alpha l_t. \quad (15)$$

Equation (15) implies

$$d_t = A \alpha k_{t+1}^\alpha l_{t+1}. \quad (16)$$

Meanwhile, substituting (11) and (12) into equation (7) implies

$$d_t = \beta A^2 \alpha (1-\alpha) k_{t+1}^{\alpha-1} k_t^\alpha (1-l_t). \quad (17)$$

Combining (16) and (17) leads to

$$k_{t+1} l_{t+1} = K_{t+1} = \beta A (1-\alpha) k_t^\alpha (1-l_t). \quad (18)$$

Substituting (12) and (18) into (9) gives

$$s_t = A(1-\alpha)k_t^\alpha[\beta - (\beta + \tau)l_t]. \quad (19)$$

Equation (8) and (12) imply

$$c_t = A(1-\alpha)k_t^\alpha(1-l_t). \quad (20)$$

Using (1), (18), (19) and (20), and substituting (12), we have:

$$k_{t+1} = \frac{\beta A(1-\alpha)}{\beta+1} k_t^\alpha, \quad (21)$$

Equation (21) determines the dynamics of the capital intensity, which drives the economic growth in this model.

### 2.3 The Growth Effect of Wage Tax

In this benchmark model, output per capita  $y_t$  is endogenously decided by capital intensity  $k_t$  ( $y_t = Ak_t^\alpha$ ). In the steady state where  $k_{t+1} = k_t = k$ , equation (21) implies

$$k^{1-\alpha} = \frac{\beta A(1-\alpha)}{\beta+1}. \quad (22)$$

Therefore, the effect of the balanced funded pension scheme on steady-state capital accumulation and output per capita can be measured by  $\frac{dk^{1-\alpha}}{d\tau}$ . It is straightforward that this funded pension scheme has no effects on steady-state capital accumulation in the benchmark model where the time discount factor is exogenously given.

**Proposition 1** When the discount factor is exogenously given, the funded pension scheme is neutral to steady-state capital accumulation ( $\frac{dk^{1-\alpha}}{d\tau} = 0$ ).

Intuitively, capital accumulation comes from voluntary saving by consumers and mandatory contribution by pension system. On one hand, mandatory contribution increases with pension contribution rate. On the other hand, voluntary saving decreases with pension contribution rate because disposal income is lower. The increase in mandatory contribution offsets the decrease in voluntary saving, and therefore funded pension scheme does not have effect on capital intensity.

It also can be concluded that funded pension scheme is neutral to steady-state output per capita which is monotonically increasing in capital intensity. Equation (22) also implies that the steady-state capital intensity increases with the constant discount factor.

### 3. Endogenous Time Preference

Section 2 discusses the benchmark model where the discount factor  $\beta$  is exogenous. Now we leave the world of exogenous discount factor. This section incorporates endogenous time preference by assuming:

$$\beta_t = f(\bar{s}_t), \quad (23)$$

where  $f'(s) > 0$ , and  $\bar{s}_t$  denotes the economy-wide average saving level. Notice that  $f' > 0$  implies decreasing marginal impatience (DMI) in saving. In other words, the consumers who hold more savings are more patient. The assumption of DMI is supported by many empirical evidences (for example, Lawrance (1991), Samwick (1998) and Ikeda et al. (2006)).

This assumption is in line with Schmitt-Grohé and Uribe (2003), Bian and Meng (2004), and Sodini (2011), which all assume that the endogenous time preference parameter is determined by aggregate per capita variable. Therefore, consumers do not internalize economy-wide average saving  $\bar{s}_t$  and take it as given. In equilibrium we have  $\bar{s}_t = s_t$  ex post, because all consumers are identical. Including saving level as the determinate of discount factor is in line with Becker and Mulligan (1997), and Stern (2006). They claim that determinants of discount factor push out current consumption.

Substituting (23) into equation (21) leads to

$$k_{t+1} = \frac{f(\bar{s}_t)A(1-\alpha)}{f(\bar{s}_t)+1} k_t^\alpha. \quad (24)$$

Hence in the steady state

$$k^{1-\alpha} = \frac{f(s)A(1-\alpha)}{f(s)+1}. \quad (25)$$

Proposition 2 demonstrates the effect of funded pension scheme on capital intensity in the presence of endogenous discount factor.

**Proposition 2** When the discount factor is increasing in economy-wide average saving, the funded pension scheme is negative to steady-state capital accumulation ( $\frac{dk^{1-\alpha}}{d\tau} < 0$ ).

This result is obtained by differentiating steady-state capital intensity with respect to proportional pension contribution rate. The proof is in the Appendix.

Intuitively, increasing pension contribution rate leads to less voluntary saving. Lower saving level causes smaller discount factor (because of DMI) which makes the consumer gives less weight to the future life. Hence the

consumer chooses to supply less labor, which causes less income and less mandatory contribution from pension. Both voluntary saving and mandatory contribution decrease, therefore capital intensity is lower.

#### 4. Concluding Remarks

This paper explains why a funded pension scheme can have a negative impact on capital accumulation. It discusses two alternative assumptions on time preference (exogenous and endogenous discount factor) in two parallel models, and compares the effect of the funded pension scheme on capital accumulation in these two models. In the benchmark model with exogenous discount factor, the funded pension scheme is neutral to capital accumulation. However, in the model with endogenous discount factor the effect of a funded pension scheme is negative.

### Appendix

#### A. Proof of Proposition 2

**Proposition 2** When the discount factor is increasing in economy-wide average saving, the funded pension scheme is negative to steady-state capital accumulation ( $\frac{dk^{1-\alpha}}{d\tau} < 0$ ).

**Proof.** I take the derivative of  $k^{1-\alpha}$  with respect to the pension contribution rate  $\tau$  in order to investigate the effect of funded pension scheme on steady-state capital intensity:

$$\frac{dk^{1-\alpha}}{d\tau} = A(1-\alpha) \frac{f'(s) \frac{ds}{d\tau} (f(s)+1) - f(s)f'(s) \frac{ds}{d\tau}}{[f(s)+1]^2} = \frac{A(1-\alpha)f'(s) \frac{ds}{d\tau}}{[f(s)+1]^2}. \quad (\text{A.1})$$

Because  $f'(s) > 0$  and  $\frac{ds}{d\tau} < 0$ ,  $\frac{dk^{1-\alpha}}{d\tau} < 0$ . Therefore, the effect of funded pension scheme on capital accumulation is negative.

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