Pension System and Time Preference in an Endowment Economy

「純粋交換経済における年金制度及び時間選好」

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Abstract

This paper incorporates the three-period overlapping generations model with a funded pension scheme and quasi-hyperbolic discounting in an endowment economy, and discusses the behavior of each generation under the general equilibrium condition. For saving instruments, there is a substitution relation between the funded pension scheme and investments. The equilibrium gross return rate on asset holdings decreases with a higher pension contribution rate and stronger present bias. The funded pension scheme deteriorates consumer welfare.

Keywords: Overlapping Generations Model, General Equilibrium, Funded Pension Scheme, Quasi-Hyperbolic Discounting

JEL Classification: D91, E21, H55

1. Introduction

A mandatory pension contribution scheme provides a savings commitment device for people who prefer immediate consumption to saving for retirement. In the case of a funded pension scheme, people's pension benefits depend on their contributions and the return rate. The aim of this paper is to investigate the endogenous gross return rate under general equilibrium conditions, and examine how it is affected by the pension contribution rate and the time preference parameter.

People with strong present bias saves at a low level (Laibson et al., 1998) and social security system provide them with compulsory savings for retirement. There are a number of studies discussing the relation between time preference, labor supply, and social security systems in a partial equilibrium. For example, Schwarz and Sheshinski (2007) examine the effects of hyperbolic discounting on the comparison of alternative social security systems.

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Diamond and Koszegi (2003) show that quasi-hyperbolic-discounting consumers may retire either earlier or later than exponential discounters, with different consumption patterns. Zhang (2013) finds that quasi-hyperbolic discounting explains why under-savers might also be early retirers. Therefore, it is of interest to ask how time preference affects consumer behavior through a funded pension system in a general equilibrium.

The main contributions are as follows. First, this paper extends the model of Diamond and Koszegi (2003) by employing endogenous labor supply and incorporating mandatory funded pension scheme. This mandatory pension scheme as a commitment device has effects on consumption level and labor supply, and hence consumer welfare. Second, this paper derives a general equilibrium model of overlapping generations in an endowment economy. Third, this paper incorporates quasi-hyperbolic discounting (Laibson, 1997) to investigate the effect of time preference on the endogenous gross return rate. Fourth, the welfare effect of the funded pension scheme is discussed under the general equilibrium condition.

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 derives the general equilibrium condition and offers certain propositions. Section 4 concludes.

2. The Model

Consider an economy populated by overlapping generations living for three periods (young, middle-aged, and old). They supply labor in-elastically when they are young and elastically when they are middle-aged. Before retirement, they receive wages at a rate of w. In the last period of their life, they do not work but exhaust all remaining resources. I assume zero population growth and that the population of each generation is one. A funded pension scheme is employed in this economy. It levies a pension contribution at a rate of $k \in [0,1]$ when the consumer is working, and when she is old, gives a pension benefit b_t to the generation born in period t (hereafter "generation t"). The time horizon is infinite.

I assume that there is only one type of goods that are unstorable. Consumers can invest them in (or borrow them from) the bank and hold assets. Variable $c_{t,i}$ (i = 1, 2, 3 here and hereafter) denotes the consumption level of generation t in each period and $s_{t,i}$ denotes the net flows (a positive one indicates outflows and a negative one indicates inflows) of assets. Variable l_t denotes the labor supply in her middle age. The inter-temporal budget constraint of generation t is

$$c_{t,3} = R_{t+1}R_{t+2}[w(1-k) - c_{t,1}] + R_{t+2}[w(1-k)l_t - c_{t,2}] + b_t , \tag{1}$$

where R_t denotes the gross return rate on asset holdings in period t. The government imposes a balanced funded pension scheme as follows:

$$b_t = R_{t+1}R_{t+2}kw + R_{t+2}kwl_t . \tag{2}$$

Consumer preference is given by

$$U_{t,j} = u(c_{t,j}) + \beta \sum_{\tau=1}^{3-j} [\delta^{\tau} u(c_{t,j+\tau})] - (\beta\delta)^{2-j} e(l_t), \qquad j = 1,2\,, \tag{3}$$

where $e(l_t)$ denotes the disutility of working in middle age. $0 < \beta \leq 1$ represents the short-run discount factor, and $0 < \delta \leq 1$, the long-run discount factor.

The preference given by (3) includes two types of consumers: exponential discounters $(\beta = 1)$ and quasi-hyperbolic discounters $(0 < \beta < 1)$. Under quasi-hyperbolic discounting, the short-run discount factor β captures the consumer's strong impatience for immediate utility. Therefore, it also can be referred to as the present-bias parameter. The behavior of an exponential discounter is considered to be the optimal outcome from the long-run perspective, because her preference is time-consistent (O'Donoghue and Rabin, 1999).

Furthermore, I make functional form assumptions by stipulating that

$$u(c_{t,j}) = \ln c_{t,j}\,, \tag{4}$$

$$\text{and } e(l_t) = \ln l_t\,. \tag{5}$$

Different responses follow different degrees of sophistication during dynamic decision making (O'Donoghue and Rabin, 1999). However, under logarithmic utility, the consumption level and labor supply of the naive consumer coincides with that of the sophisticated one. Therefore, differentiating between the two types of consumers is not necessary. The functional form assumption of logarithmic utility enables us to focus on time preference parameters rather than the degree of sophistication.

Budget constraint (1), preference (2), and functional form assumptions (4) and (5), jointly determine the consumption level and labor supply of generation t.

$$c_{t,1} = \frac{w}{(1+\beta\delta^2)}\,, \tag{6}$$

$$c_{t,2} = \frac{\delta w R_{t+1}}{(1+\beta\delta^2)}\,, \tag{7}$$

$$c_{t,3} = \frac{\beta\delta^2 w R_{t+1} R_{t+2}}{(1+\beta\delta^2)}\,, \tag{8}$$

$$\text{and } l_t = \frac{\delta R_{t+1}}{(1+\beta\delta^2)}\,. \tag{9}$$

3. General Equilibrium in the Endowment Economy

3.1 Equilibrium Gross Return Rate on Assets Holdings

I assume the consumers are homogeneous in time preference. In this endowment economy, consumers have to invest their savings in the bank given the nature of goods (unstorable), and hence, the gross return rate on asset holdings R_t is endogenously determined. The general equilibrium condition in period $t+1$ reads as

$$s_{t-1,3} + s_{t,2} + s_{t+1,1} = 0 \quad .(10)$$

Because $s_{t+1,1} = w(1-k) - c_{t+1,1}$, $s_{t,2} = (R_{t+1} - 1)s_{t,1} + w(1-k)l_t - c_{t,2}$, and $s_{t-1,3} = (R_{t+1} - 1)[R_t s_{t-1,1} + w(1-k)l_{t-1} - c_{t-1,2}] + b_{t-1} - c_{t-1,3}$, substituting consumption level and labor supply leads to the equilibrium gross return rate:

$$R_t = \frac{\beta\delta^2 - (1+\delta+\beta\delta^2)k}{\delta(1-k)} . \quad (11)$$

The endogenous gross return rate is time-invariant $(R_t = R)$. Further, equation (11) implies that if and only if $0 \le k < \dfrac{\beta\delta^2}{1+\delta+\beta\delta^2}$, the gross return rate on assets is positive.

Proposition 1 discusses how equilibrium in an endowment economy relates to the funded pension scheme.

Proposition 1 In an endowment economy with an endogenous gross return rate on asset holdings, equilibrium exists if and only if $0 \le k \le \dfrac{\beta\delta^2}{1+\beta\delta^2}$ $(R \ge 0)$, and the gross return rate decreases with a higher pension contribution rate rises $(\dfrac{dR}{dk} < 0)$.

Proposition 1 states that a sufficiently low pension contribution rate leads to a non-negative gross return rate on asset holdings and equilibrium among generations in each period.

In this endowment economy, consumers cannot store their savings. They may save for retirement by investing savings in a bank and receiving repayments, or making contributions to the funded pension system and receiving pension benefits. In other words, there is a substitution relation between investments and the funded pension scheme. Therefore, the gross return rate on asset holding decreases with higher pension contribution rates.

When the gross return rate on assets is positive, each young-generation consumer invests her savings in the bank and receives repayments in her middle-aged and old period. For each period, the amount invested by the young generation provides the repayments received by the middle-aged and old generations.

When the gross return rate on assets is zero, each generation invests savings when she is young and receives repayments in her middle age. The pension benefit covers her consumption when she is old. The net flow of the old generation is zero and only the middle-aged generation gets repaid. In these contexts, the equilibrium in this endowment economy maintains both intra- and inter-generational balances.

In the absence of a funded pension scheme, i.e., $k = 0$, instrument in banks and their repayments serve as the sole saving instrument. The old generations receive no pension benefit but to rely on the remaining assets. Therefore, the strong desire of young generation to save for old period pushes up the gross return rate to a high level.

As the pension contribution rate increases, the funded pension system becomes another saving instrument. Old generations receive not only repayments from the bank, but also pension benefits from the funded pension system. Therefore, the decreasing demand for bank investments among young generations drags the gross return rate down.

In the extreme case of $k = \frac{\beta\delta^2}{1+\delta+\beta\delta^2}$, the gross return rate equals to zero. The pension contribution rate is so high that the pension benefits itself can cover the old generations' consumption, and the young generations are no compelled to save. As the pension contribution rate increases beyond $\frac{\beta\delta^2}{1+\delta+\beta\delta^2}$, the equilibrium is broken.

Notice that Proposition 1 implies that the equilibrium gross return rate is less than 1. In other words, young generations invest their savings in the bank even though they have to bear a loss in their asset holdings.

3.2 Time Preference

Consumer preference given by (3) implies different time preferences: exponential discounting ($\beta = 1$) or quasi-hyperbolic discounting $(0 < \beta < 1)$. A lower present-bias parameter β describes stronger impatience prompting immediate consumption, and leads to a lower level of saving. Proposition 1 implies that the existence of equilibrium in this endowment economy is related to time-preference parameters. Proposition 2 states the manner in which the gross return rate relates to time preference.

Proposition 2 In an endowment economy with an endogenous gross return rate on asset holdings, the gross return rate decreases with stronger present bias (i.e., a lower β).

Stronger present bias leads to lower levels of saving, lower demand for investment by young generations, and hence, a lower gross return rate on asset holdings. Furthermore, a lower present-bias parameter implies that realizing equilibrium in the endowment economy is more difficult.

3.3 Welfare Effect of Funded Pension System

This sub-section investigates the welfare effect of a funded pension system. In this general equilibrium model of an endowment economy, the gross return rate is endogenously determined, while the equilibrium depends on the pension contribution rate. The funded pension system can adjust consumer behavior by controlling the pension contribution rate.

Here, ex-ante welfare is evaluated at the point of birth of the consumer. I evaluate the consumer's well-being from a long-run perspective, namely under exponential discounting:

$$W_t = u(c_{t,1}) + \delta u(c_{t,2}) + \delta^2 u(c_{t,3}) - \delta e(l_t)\,. \tag{12}$$

The government aims to optimize consumer welfare by selecting the optimal pension contribution rate. This welfare effect is investigated by differentiating welfare with respect to the pension contribution rate. Proposition 3 shows how the funded pension system affects consumer welfare.

Proposition 3 In an endowment economy with an endogenous gross return rate on asset holdings, a funded pension scheme is welfare-deteriorating when the gross return rate is positive.

The proof of Proposition 3 is in the Appendix.

The funded pension scheme affects welfare through the endogenous gross return rate. Recalling from Proposition 1, an increase in the pension contribution rate leads to a decrease in the gross return rate. On one hand, a decrease in gross return rate implies lower consumption level in middle-aged and old period because of lower return on assets. On the other hand, a decrease in the gross return rate leads to lower pension benefits owing to lower labor supply in the middle age. Therefore, the lower gross return rate caused by higher pension contribution rate decreases the efficiency of savings investments by young generations and therefore, leaves consumers worse off.

4. Concluding Remarks

By incorporating the overlapping generations model under quasi-hyperbolic discounting with a funded pension scheme, this paper shows that the equilibrium condition depends on the pension contribution rate and time preference parameters. The endogenous gross return rate decreases with a higher pension contribution rate and a lower present-bias parameter. Consumer welfare decreases as the pension contribution rate increases.

Appendix

A. Proof of Proposition 3

Proof. I take the derivative of welfare with respect to the pension contribution rate k in order to investigate the welfare effect:

$$\frac{dW_t}{dk} = u'(c_{t,1})\frac{dc_{t,1}}{dk} + \delta u'(c_{t,2})\frac{dc_{t,2}}{dk} + \delta^2 u'(c_{t,3})\frac{dc_{t,3}}{dk} - \delta e'(l_t)\frac{dl_t}{dk}. \tag{A.1}$$

Substituting first-order conditions, consumption level and labor supply leads to:

$$\frac{dW_t}{dk} = \frac{2\beta\delta^4 w}{1+\beta\delta^2}u'(c_{t,3})R\frac{dR}{dk}, \tag{A.2}$$

where $\frac{dR}{dk} < 0$ because of Proposition 1. Therefore, for a positive gross return rate we have $\frac{dW_t}{dk} < 0$. Thus, the funded pension scheme is welfare-deteriorating.

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