Leibniz on Derivative Force

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In this paper I wish to examine the nature and role of "derivative force" (G5 156 = NE 2.21.1) in the later philosophy of Leibniz. In the first part of the paper, I discuss two characterizations of derivative force, and argue that there are two different derivative forces in *prima facie*. In the second part of the paper, I attempt to specify Leibniz's theory concerning the ontological status of the physical derivative forces. This leads finally to my hypothesis that the sum of degrees of perfection is conserved in a body, and to a discussion of several difficult problems in this explanation of physical derivative force.

1. Two Characterizations of Derivative force

Leibniz emphasized that a certain quantity is conserved in the course of nature; i.e. a vital force (vis viva) measured by mv^2. In fact, Leibniz took many physical forces (kinetic energy, impetus, resistance, etc.) to be derivative forces involved in bodies. In this context, derivative force is supposed to belong to an aggregate of substances since bodies are aggregates of substances in Leibniz's metaphysics (LA 170 = G2 135 etc.).

> The forces which arise from mass and velocity are derivative and belong to aggregates, that is, to phenomena. (AG 176 = G2 251 1703.6.20)

However, there is a serious problem. That is, a definition of derivative force suggests that it is totally heterogeneous to physical forces such as kinetic energy and resistance. Actually many commentators point out that Leibniz introduces these two notions of derivative force that seem to be distinct. Let us see the definition. For example, the next passage suggests that derivative force is identical to a present perceptual state of a simple substance.

> Derivative force is itself the present state when it tends toward or preinvolves a following state, as every present is great with the future. But that which persists, insofar as it involves all cases, contains primitive force, so that primitive force is the law of the series, as it were, while derivative force is the determinate value which distinguishes some term in the series. (L 533 = G2 262 1704.1.21)

In this context, derivative force belongs to a single substance rather than an aggregate of substances.

Also, derivative force is regarded as "modification (modificatio)" and a single substance has its modification so long as it continues to change:

> And indeed, derivative forces are only the modifications and resultants of the primitive forces. (AG 176 = G2 251 1703.6.20)

> You speak as if you do not understand what I intend when I say that derivative forces are mere modifications and that the active cannot be a modification of the passive. …For you interpret me as denying that derivative forces are active, since you say, "Therefore, I do not see why these (derivative) forces are not active." But so far am I from denying that they are active that I infer, from the very fact that they are active and yet modifications, that there is something primary and active of which they are modifications. (L 533 = G2 262 1704.1.21)

To understand this notion of derivative force adequately, it is better to understand the notion of primitive force since it is defined in relation to the primitive force.

> And when I speak of the primitive force that persists, I don't understand the conservation of total motive power about which we were once concerned, but the entelechy that always expresses that total force, as well as other things. (AG 176 = G2 251)

> [B]ut I think that it is obvious that primitive forces can be nothing but the internal strivings of simple substances, strivings by means of which they pass from perception to perception in accordance with a certain law of their nature, and at the same time harmonize with one another, representing the same phenomena of the universe in different ways, something that must necessarily arise from a common cause. (AG 181 = G2 275).

Primitive forces are identified with monads or simple substances.[i] They are similar to laws of series that generate all the particular terms, while derivative forces are similar to these particular terms. Thus derivative forces are associated with modifications of substances in this context, and primitive ones are associated with substances.

Now we have seen the notion of derivative force in two different contexts. On the one hand, derivative forces are said to exist in bodies and typical examples of them are a kinetic energy and resistance. On the other hand, derivative forces are supposed to exist in substances as modifications. Considering that derivative force is discussed in these two contexts, Martial Gueroult concludes that the notion of derivative force has a mixed character (Gueroult 1934, p.199).

2. The Ontological Status of a Physical Derivative Force

To be sure, Gueroult sagaciously demonstrated a significant problem concerning the notion of force in Leibniz as early as in the former half of the 20th century. However, he did not try to explain the notion of derivative force univocally. Instead, he seems to leave the problem as something inherent to Leibniz's metaphysics. Now our main issue is to explain the ontological status of a physical derivative force. Through this research, we will see whether it is identical to the modifications of simple substances.

Leibniz states that "the forces which arise from mass and velocity" belong to aggregates or phenomena (AG 176 = G2 251). If the velocity is an internal phenomenon for a perceiver, then the forces in this case are also phenomenal.[ii] Then it seems to be difficult to identify them with the modifications of simple substances in a body.[iii] When Leibniz talks about the conservation of forces, what kind of force does he discuss? Does he discuss a real force that exists in a body? Or he just refers to the law of nature which phenomenal bodies observe?

The latter is a phenomenalistic explanation of the conservation of forces. All phenomenal bodies appear to us in such a way that all the forces in bodies seem to be conserved.[iv] For example, suppose two billiard balls that elastically collide to each other. These balls appear as if their inner forces were conserved. One of the problems of this interpretation is that it may not be coherent with the claim that forces are more real than locomotion. If forces are not conserved, then forces are as phenomenal as locomotion. However, Leibniz states that forces are more real than relative motions:

> The discussion is not long, and the matter is of great importance, not only for mechanics but also in metaphysics, for motion in itself separated from force is merely relative, and one cannot establish its subject. But force is something real and absolute, and since its reckoning is different from that of motion, as I demonstrate clearly, one must not be surprised that nature preserves the same quantity of force and not the same quantity of motion. (LA 167 = G2 133)

> Regarding physics, one must understand the nature of force, quite different from movement, which is something relative. That one must measure this force by the quantity of the effect. That there is an absolute force, a directive force and a relative force. That each of these forces is preserved in the same degree in the universe, or in each machine not communicating with others, and that the last two forces, taken together, make up the first or absolute force. But the same quantity of motion is not preserved, since I show that otherwise perpetual motion would be discovered, and that the effect would be more powerful than its cause. (LA 172 = G2 137)

We can understand that the conserved forces cannot have such a weak ontological status from these two passages in a letter to Arnauld. The conserved forces can be measured by their effects, and these are regarded as the causes of the effects or changes of positions. Since the causes are supposed to have higher ontological statuses than effects, it seems to be difficult to argue that forces are just appeared to be conserved.

Another interpretation might be like this: when Leibniz talks about the conservation of forces, he only considers primitive forces. In any simple substance, its primitive force is constant. Even though a simple substance perpetually changes its perceptual states, the law of series of perceptual states, that is, the primitive force does not change (Adams 1994, pp.314-6 cf. Garber 1985, p.88 etc.). Thus all the primitive forces are "conserved" in nature. One of the problems of this interpretation is that it may contradict the fact that Leibniz is thinking about the forces of bodies when he refers to the conservation of forces. For example, Leibniz states that the forces of bodies are conserved after a collision. Also, Leibniz clearly states that "even a derivative force is conserved in nature" (G3 437). Therefore, we need to consider the conservation of derivative forces rather than primitive ones.

Now we have seen two interpretations of the conservation of forces and difficulties with them. Is there another way of explaining these difficulties? Now we should consider suggestions of Robert Merrihew Adams, who understands the preceeding studies of Gueroult and tries to explain derivative forces of bodies in terms of modifications of simple substances (Adams 1994, pp.378-99). Indeed, several passages suggest that Leibniz does not have two different concepts of derivative force. In other words, Leibniz clearly declares that physical derivative forces are modifications of primitive forces.

> [T]he derivative or accidental force that cannot be denied to bodies in motion must be a modification of the primitive [force], as shape is a modification of extension. Accidental forces cannot occur in a substance without essential force, for accidents are nothing but modifications or limitations, and cannot contain more perfection or reality than the substance. (G3 457 1703.3.22; Adams 1994, p.378)

In this passage, a derivative force that belongs to a body (not a substance but an aggregate of substances) is taken to be modifications of primitive forces. Therefore, we should not suppose that there are two different derivative forces, that is, physical derivative forces and derivative forces as modifications. These two are supposed to be the same (Adams 1994, p.382).

However, Adams does not wholly accept this explanation. In fact, Adams suggests that the modifications of simple substances "express" the physical forces of organic bodies, and he realizes that the expression does not imply the identification. That it to say, even if there is a constant and regular correspondence, this does not mean these two are actually one and the same. Adams seems to

talk about a phenomenal organic body that exists in a perceiver since he thinks that even an aggregate of simple substances exists in a perceiving mind (Adams 1994, pp.246-7). For example, when I perceive the organic body of a kitten, that body and its derivative force exist in my mind (Adams 1994, pp.286-7).

On the other hand, I will consider another kind of force in a body in this paper.[v] By the term "physical derivative force" or "physical force," I am referring to a derivative force that exists in a particular group of monads or simple substances which are supposed to found the reality of a particular body.

To endorse this notion of physical derivative force, I have to answer a question which Adams proposes. Adams states that one organic body can exist in all the created simple substances in the universe. For example, not only I but my friend in the same room may consciously see the organic body of the kitten. In that case, it seems to be reasonable to say that the organic body of the kitten also exists in the mind of my friend, too. Also, according to Leibniz's metaphysics, any simple substance expresses or perceives any body in the universe. So some people in China or India at least unconsciously perceive the organic body of the kitten, and even bare monads that found the reality of inorganic bodies perceive it. Thus the organic body exists in any simple substance in the universe.

Adams' claim is reasonable and I have to admit that any simple substance in the universe has any organic body in its perception as an internal phenomenon. Any created substance expresses all the other substances in the universe. And a created simple substance perceives another created substance through its organic body: I perceive the kitten through its organic body, and also perceive a dog through its own body.

But it is also true that Leibniz states that a certain monad or a particular group of monads are the "cause" of an event. To be sure, in the realm of internal phenomena, a perceiving monad causes all the events in its perception since any monad can produce all the internal phenomena in it (W 247 = G4 260). But according to Leibniz, one monad has a stronger tendency to actualize the event than other monads.[vi] That is, one monad has a more distinguished or distinct perception of the event than the others (Adams 1994, p.318). For example, when a phenomenal body is moving in a certain velocity, it could be possible that a certain group of monads have more distinguished tendency to realize that locomotion than the others. Thus we can pick out a group of monads related to the event.

Or another explanation may be possible. A dominant monad expresses or corresponds to its organic body. The relationship is of one-to-one mapping (Adams 1994, p.286). Then, when we measure the kinetic energy of a body, we associate it with that body. And since that body is either an organic body or an aggregate of organic bodies, we can assign one or many dominant monads to that phenomenal body. Also, a dominant monad has many subordinate monads (G2 252). Therefore, we can assign these monads to that phenomenal body, too. Thus we can pick out a particular group of monads when we see a certain phenomenal body moving and changing its place.

Now that we have a particular group of simple substances or monads, let us consider the perceptual states of these simple substances selected from all the simple substances in the universe. Then it is reasonable to state that these perceptual states are modifications of a particular group of simple substances since each perceptual state is a modification of one simple substance.

To be sure, there are some objections to this line of interpretation of derivative force in a body. For example, Paul Lodge argues that we cannot identify the derivative forces of organic bodies with the modifications of dominant monads (Lodge 2001, pp.723-4). As I pointed above, Adams suggests that the derivative forces or organic bodies *express* or *represent* the primitive forces of their dominating monad through the mediation of the appetitions, or the derivative forces of that monad. But the relation of expression is not that of identity. Considering this, Lodge proposes another explanation of physical derivative force as the modifications of simple substances.

Indeed, if there are more persuasive explanations, then we should consider them. So let us see Lodge's alternative. He suggests that the relationship between the primitive forces of monads and the derivative forces of their organic bodies is a relation of modification, "where modification is understood in the Platonic sense" (Lodge 2001, p.726). One example of this relation is that between God and creatures. "Modifications need not inhere in the things they modify, they need only be limitations of a substance, in the sense that they manifest or instantiate the essence of the substance in an inferior way" (Lodge 2001, p.726).

But even if we concede that this notion of modification is coherent, we can doubt whether Leibniz accept this theory of modification in the late period. In fact, Leibniz suggests that a modification exists in a substance. For example, Leibniz states that "we ourselves always produce within us" and "phenomena are simple new transitory modifications of our souls" (L 626 = G6 591). Leibniz seems to have the Cartesian notion of modification. According to the Cartesian theory, sensation, idea and will can exist only in a mind or soul, since all of these are modes of a mind. So it is natural to suppose that Leibniz's notion of modification is similar to that of Descartes rather than Neo-Platonians.

These considerations suggest that physical derivative forces should be explained by the Cartesian notion of modification. In other words, physical derivative forces are to be explained in terms of a modification of single substance since a modification of substance cannot exist in many substances at the same time.

Then, can we explain a physical force by modifications of non-extended simple substances? The important point is that a physical force seems to postulate the existence of physical motions in a space, while simple substances are not extended and cannot spatially move. This is a difficult problem, but I think the notion of absolute motion can be a key to understand the issue. Leibniz states that locomotion is relative since a moving body is at rest from another frame of reference.

> For if we consider only what motion contains precisely and formally, that is, change of place, motion is not something entirely real, and when several bodies change position among themselves, it is not possible to determine, merely from a consideration of these changes, to which body we should attribute motion or rest, as I could show geometrically, if I wished to stop and do this now. (AG 51 = DM 18)

But we should not immediately conclude that Leibniz does not take any motion to be real. On the contrary, he suggests the existence of absolute motions in the fifth letter to Clark:

> However, I grant there is a difference between an absolute true motion of a body and a mere relative change of its situation with respect to another body. For when the immediate cause of the change is in the body, that body is truly in motion, and then the situation of other bodies with respect to it will be changed consequently, thought the cause of that change is not in them. (AG 341 = G7 404)

"The immediate cause of the change" seems to refer to the primitive forces of simple substances in a body or aggregate. When these primitive forces act, there are inner changes in all the simple substances in the body. So the term "absolute true motion" should refer to these inner or perceptual changes. So it is possible to suppose that Leibniz is actually considering this kind of absolute motion when he talks about a force of a body. If it is true, then we may be able to explain a force of a body by absolute motions or perceptual changes of simple substances in that body.

Here we should consider the notion of "cause" of an event again. As I pointed out before, Leibniz states that a certain monad or a particular group of monads are a cause of an event: one monad has a more distinguished or distinct perception of the event than the others. So when we see a phenomenal body is moving, we can think about the absolute motion of a particular group of monads regarded as the cause of the phenomenal locomotion. Simple substances with absolute motions are supposed to have more distinguished tendencies to realize the event than the others.

Now we have seen the notions of modification and absolute motion in Leibniz. When Leibniz talks about the physical derivative force, he may be thinking of the modifications of simple substances that are regarded as inner absolute motions of them. In the next section, I will suggest my hypothesis to explain the conservation of physical derivative force on this line and its intrinsic problems.

3. The Conservation of the Sum of Perfection

We have seen that there is an absolute motion when we see a phenomenal extended body

moving. Then we can suppose that when a body has an absolute motion, every simple substance in the body has its inner change. Thus it is possible to assign a derivative force as a "present state (status praesens) (G2 262 1704.1.21)" to an aggregate of substances. In this case, we are obliged to reduce the present state of a body to the present states of many simple substances that constitute the body or aggregate. In other words, we should translate the fact that an aggregate has a derivative force to the statement that a simple substance **a** has a certain status **Sa,** another simple substance **b** has a status **Sb**, another simple substance **c** has a status **Sc**.... and all of these simple substances belong to the same aggregate.

Now I will propose my hypothesis about how the derivative force of an aggregate of simple substances can be conserved. It is true that each simple substance continues to change and never remains in the same state. For instance, let us name the state of simple substance **a** at the moment $\mathbf{t_1}$ $\mathbf{Sa_1}$, and the state of simple substance **b** at the same moment $\mathbf{Sb_1}$. Also, let us call the state of simple substance **a** at the different moment $\mathbf{t_2}$ $\mathbf{Sa_2}$, and the state of simple substance **b** at the same moment $\mathbf{Sb_2}$. In this case, $\mathbf{Sa_1}$ and $\mathbf{Sa_2}$ are different, and so are $\mathbf{Sb_1}$ and $\mathbf{Sb_2}$. Then why a constant quantity is conserved in the aggregate that contains both **a** and **b**? The notion of perfection can be a key to auswer this question. Leibniz talks about perfections of various entities. He often refers to the perfections of God (G5 286 = NE 3.4.12, G5 418 = NE 4.10.7, G5 422 = NE 4.10.9 etc.). But simple substances created by God also have their own perfections (G4 564). Each simple substance continues to change its perfection. In other words, its degree of perfection is not constant. For example, Leibniz often states that pleasures come from perfections and pains come from imperfections (G5 180 = NE 2.21.41 etc.). A soul feels greater pleasures when its perfection increases, and it feels greater pains when its perfection decreases (G5 187 = NE 2.21.58 cf. G5 195). In this case, it is possible to consider the degree of perfection of a simple substance at a certain moment. Also, in a letter to Bourget, Leibniz talks about the sum of perfections of all created substances in the universe at a certain moment (G3 592). And he states that even if a number of created substances is infinite, the universe does not necessarily have a supreme perfection. For instance, although the sum of series 1+1/2+1/3+1/4+1/5.... and the sum of series 1+1+1+1+1... are both infinite, the latter is infinitely larger than the former. This discussion is plausible since we can consider a degree of perfection of a created substance at a certain moment.

Therefore, for example, even if $\mathbf{Sa_1}$ is different from $\mathbf{Sa_2}$ and $\mathbf{Sb_1}$ is different from $\mathbf{Sb_2}$, it is possible to suppose that a certain quantity of derivative force has been conserved from $\mathbf{t_1}$ to $\mathbf{t_2}$ when the sum of the degrees of perfection at $\mathbf{t_1}$ and the sum of the degrees of their perfection at $\mathbf{t_2}$ are the same (cf. Adams 1994, p.389).

Then the next question arises: it may possible to consider increases and decreases of perfection of a whole aggregate since each simple substance increase or decrease its own perfection in that aggregate. However, since simple substances do not act on each other, a body as an aggregate of

substances actually cannot receive any forces from outside. Indeed, Leibniz agrees to this idea. A common sense may tell us that a body gains its force from another body, but it is actually false. For example, Leibniz states that the passions of a body arise from internal and spontaneous force. The body may have an external occasion, but external forces do not cause its motion (GM6 251). And the next passage clearly suggests that an aggregate of simple substances does not gain any external force.

> Meanwhile, I find it to be true in phenomena as well, and in derivative forces, existing that masses do not so much give other masses new force as determine the force already in them, So that a body drives itself away from another by its own force, rather than being propelled by the other. (Lodge 263 = G2 251)

From this passage, we can understand that a body or mass as an aggregate of simple substances actually acts by its own force. Thus not only individual substances but aggregates continue to change by their internal force. Exchanges of force appear to exist, but actually do not. Further, as Adams points out, Leibniz again argues that bodies never exchange forces in the letters to Christian Wolff (Adams 1994, pp.383-6).

> It should be known, however, that forces do not cross from body into body, since any body whatever already has in itself the force that it exerts, even if it does not show it or convert it into motion of the whole prior to a new modification. For example, when a ball that is at rest is struck by another, it is moved by an implanted force, namely by elastic force, without which there would be no collision. Moreover, the Elastic force in the body arises from an internal motion invisible to us. And the Entelechy itself is modified corresponding to these mechanical or derivative [forces]. Therefore it can be said that force is already present in every body, and it is determined only by modification. (LW 131; Adams 1994, p.383)

> You ask how the primitive force is modified, for instance when the motion of heavy [bodies] is accelerated by falling. I reply that the modification of the primitive force that is in the Monad itself cannot be explained better than by expounding how the derivative force is changed in the phenomena. For what is exhibited in phenomena extensively and mechanically is in Monads in a concentrated and vital way.... And what is exhibited Mechanically or extensively through the reaction of what resists and the restoration of what was compressed is concentrated dynamically and monadically (as I have already said) in the Entelechy itself, in which is the source of the mechanism and the

> representation of [things] mechanical; for phenomena result from Monads (which are the sole true substances). And while [things] mechanical are determined by external circumstances, for that very reason [eo ipso], in the source itself, the primitive Entelechy is modified harmoniously through itself, since it can be said that the body has every derivative force of its own from itself. (LW 138-9; Adams 1994, p.385)

These passages clearly suggest that a body has its own derivative force and continues to modify it only by itself. Can an aggregate of simple substance determine its future state without external actions? First, changes in collisions are determined by the law of collision. In fact, Leibniz thinks that a primitive force has a role to determine the whole series of change (G2 262 1704.6.30 etc.). Therefore, it is possible to the state of a body before a collision as modifications of primitive forces take. Of course, primitive forces belong to individual substances rather than aggregates. But primitive forces that belong to individual substances bring about their states. And if a state of an aggregate is identical to the states of all members in the aggregate, it is possible to suppose that primitive forces of individual substances determine the present state of an aggregate.

Before examining intrinsic problems of my hypothesis, I will see some results of this explanation of physical derivative force in terms of perfections of simple substances. For example, suppose that there is a perfectly elastic collision of two billiard balls and the sum of mv^2 remains the same through the collision. If a ball (as an aggregate of simple substances) has a larger speed or kinetic energy after the collision, a member of the aggregate is likely to increase its perfection. At least, the sum of degrees of perfection must have increased. On the other hand, if a ball loses its speed or kinetic energy, then a member is likely to decrease its perfection.

So far we have attempted to propose a theory to explain physical forces in terms of modifications or statuses of simple substances. However, it is not easy to conclude that we have sufficiently solved the problem of derivative force because there are at least several difficulties in this theory of derivative force.

Adams already points out some problems concerning derivative passive forces. In fact, Adams admits that "there are indeed serious difficulties in the quantitative treatment of Leibnizian primary matter" (Adams 1994, p.395). First, if a resistance is a physical or corporeal expression of the confusedness of a perception, and if a resistance is proportional to the size of a body, then it seems that an elephant is less perfect than a mosquito. But we see that an elephant is more intelligent than a mosquito. Second, if we suppose that the quantity of the primary matter of an inorganic body is the sum of the quantities of the primary matter belonging to all the substances whose organic bodies are contained in the inorganic body, then it is difficult to measure it by the volume of the inorganic body. Every organic body has other organic bodies as parts. Therefore, the volume of an organic body is

counted not only as the organic body of a dominant monad but as an aggregate of organic bodies of subordinate monads. Since these subordinate monads have other monads further subordinate to them, the sum of the volumes of the organic bodies must be infinitely larger than the volume of the organic body of the dominant monad. Third, it is difficult to understand the notion of primary matter univocally. If we take "primary matter" to refer to primitive passive force, then it is difficult to measure it quantitatively since a primitive passive force is supposed to be indivisible.

In this paper, I have mainly considered the relationship between a kinetic energy and modifications of substances. However, even in the case of kinetic energy, we are obliged to admit that there are some difficult problems similar to the second problem suggested by Adams: perhaps the most crutial problem is that there are an infinite number of simple substances in any aggregate.

Before considering this difficulty, I assue that a number of substances in an aggregate must be infinite according to Leibniz's metaphysics. When Leibniz talks about an infinite division of body, he tries to state that a body is an aggregate of infinitely many simple substances.

Let us see the text concerning an infinite division of body. When Leibniz compares "ideal things (idealia)" to "actual things (actualia)," "real substances" are regarded as typical examples of actual things (G2 282 1706.1.19).

> But continuous quantity is something ideal, something that pertains to possibles and to actual things considered as possible. The continuum, of course, contains indeterminate parts. But in actual things nothing is indefinite… As long as we seek actual parts in the order of possibles and indeterminate parts in aggregates of actual things, we confuse ideal things with real substances and entangle ourselves in the labyrinth of the continuum and inexplicable contradictions. (AG 185 = G2 282 1706.1.19)

Also, the next passage, in which Leibniz refers to an actual division, suggests that many independent substances belong to one body or aggregate:

> [S]ince the actual world does not remain in the indifference of possibility, since it comes from effective divisions or multitude, whose results are the phenomena which are present and various in the least parts. (G4 568 1702)

Phenomena "come from (étant venu)" a multitude. And the "result" of actual multitude is supposed to be a phenomenon. These two expressions suggest that what brings about phenomena has a higher ontological status than that of phenomena. Then it is natural to suppose that an individual contained in an actual multitude is not a phenomenon but a simple substance. The claim that a body as an aggregate of substances is actually infinitely divided implies that members of the aggregate are

independent of each other and the number of members is infinite:

> But I think that the contrary is to be concluded, namely, that we must return to indivisible unities as the *primary constituents* in corporeal bulk, i.e., in the corporeal things to be constituted. Unless perhaps you mean "it is rightly concluded that bodily masses themselves are not indivisible unities." I admit this, but we are not concerned with it. Certainly bodies are always divisible — indeed, they are actually subdivided as well — but not their constituents. (Lodge 301 = G2 267-8)

In this passage, a body is taken to be something that is construed by unities or simple substances. Every part of aggregate is itself an aggregate of substances, which makes the number of substances in an aggregate infinite.

Also, in the *New Essays*, Leibniz emphasizes that a body is actually infinitely divided in the context of his criticism against atomism:

> But the colour yellow is a reality, all the same, like the rainbow. Also we are apparently destined to achieve a much higher state [of knowledge] than we are now in, and may even go on rising for ever, since corporeal nature does contain elementary particles. If there were atoms, as our author appeared elsewhere to believe that there are, perfect knowledge of bodies could not be 'beyond any finite being.' Lastly: if our eyes became better equipped or more penetrating, so that some colours or other qualities disappeared from our view, others would appear to arise out of them, and we should need a further increase in acuity to make them disappear too; and since matter is actually divided to infinity, this process could go on to infinity also. (RB 219 = G5 203-4 = NE 2.23.12)

The preestablished harmony of soul and body helps to understand the reason why the number of simple substances is infinite. According to the preestablished harmony of soul and body, an organic body corresponds to one simple substance (the dominant monad of that organic body). Let us consider a number of organic bodies in an inorganic body. We might be tempted to think that that number is finite since even a tiny organic body must have an assigned volume. However, Leibniz suggests that a number of organic bodies is infinite. In a letter to John Bernoulli, Leibniz compares an inorganic body to a pond in which many fish live (AG 167-8 = GM3 542). A pond is divided into individual fish and water. A fish has its own organic body, but water does not. But we can further find many tiny fish in the water and this series never ends.

Then why does a body have a certain quantity of derivative force? Why can a body have a finite amount of derivative force while the number of simple substances in it is infinite? One

possible answer is that even though each simple substance has an assigned quantity of perfection at a moment, the sum of all quantities of perfection in an aggregate converges to a finite quantity. Consider the series: 1 + 1/2 + 1/4 + 1/8.... This series converges to 2. But it should be noted that this model depends upon some aberrant presuppositions. First, it seems to be miraculous that any simple substance has such a harmonious relationship to others in an aggregate. Suppose that the highest degree of the perfection of a simple substance in an aggregate is 1. Then the second highest must be 1/2 and there is no simple substance whose degree of perfection is more than 1/2 and less than 1. In the same way, the third highest degree must be 1/4 and there is no substance whose degree of perfection is more than 1/4 and less than 1/8. But it seems to require a huge luck if an infinite number of simple substances keep this harmonious relationship altogether. Second, even if we concede that a body has a certain quantity of derivative force at a moment, it is true that the parts of the body perpetually change. Some parts go out and others come and stick to the body. In that case, it is unnatural to suppose that the degrees of the perfection of these new parts also make up a part of convergent series.

Then is there any other alternative to explain why a body has a finite amount of derivative force? Another possible answer is that an infinite number of degrees of perfection are integrated to a finite quantity. Indeed, Leibniz was a great pioneer of differential and integral calculus. Therefore, it may possible to suppose that Leibniz was trying to associate the sum of derivative forces with integration. However, it should be noted that a degree of perfection of simple substance is regarded as infinitesimal in this case. Leibniz points out that an infinitesimal is a fiction, which seems to be incoherent to the claim that the degree of the perfection of a simple substance is infinitesimal (GM4 110 1702.6.20). If a simple substance "actually" has an infinitesimal degree of perfection, why an infinitesimal quantity should be a fiction?

These problems are hard to solve, and in fact Leibniz seems to propose no answer to these problems at least in the philosophical works edited by C.I. Gerhardt. I should research the mathematical texts and other philosophical works from now on, but now what I could do is to note the significant difficulties.

Conclusion

In summary, I have argued that Leibniz is not attempting to suggest two different derivative forces. On the contrary, Leibniz has tried to reduce physical derivative force to modifications of substances. Considering his attempt to explain derivative forces univocally, I suggested my hypothesis based on the fact that each created simple substance has its own degree of perfection at a certain moment. According to this hypothesis, the degrees of perfection of created simple substances can be sum up to a certain quantity. Then the primary problem of the notion of derivative force does

not consist in its ambiguity. Rather, one of the main problems is to explain why an infinitely many derivative forces can converge to a finite quantity. Granted that there are several difficulties concerning the reduction of physical force, it is still worthwhile to consider a physical derivative force in a particular group of monads or simple substances in order to overcome the ambiguity of derivative force.

Abbreviation :

AG = *G. W. Leibniz: Philosophical Essay*s. Ed. and trans. Roger Ariew and Daniel Garber. Indianapolis: Hackett, 1989.

C = *Opuscules et fragments inédits de Leibniz.* Ed. Louis Coutulat. Paris : Presses Universitaires, 1903 ; repr. Hildesheim : Georg Olms, 1961.

DM = *Discourse on Metaphysics.* Cited by section number.

G = *Die philosophischen Schriften von G. W. Leibniz.* Ed. C. I. Gerhardt. Berlin: Weidmann, 1875-1890. Reprint, Hildesheim: Georg Olms, 1978. Cited by volume and page.

GM = *Leibnizens Mathematische Schriften.* Ed. C. I. Gerhardt. Berlin: A. Asher, and Halle: H.W.Schmidt, 1849-1863. Cited by volume and page.

L = *Philosophical Papers and Letters.* Trans and ed. Leroy E. Loemker. Chicago: University of Chicago Press. 1956.

LA = *The Leibniz-Arnauld Correspondence.* Trans. and ed. by H. T. Mason, with an introduction by G. H. R. Parkinson. Manchester: Manchester University Press, 1967.

Lodge = *The Leibniz - De Volder Correspondence.* Trans and ed. by P.Lodge New Haven : Yale University Press. 2013.

LW = *Briefwechsel zwischen Leibniz and Christian Wolff.* Ed. By C. I. Gerhardt. Halle: H. W. Schmidt, 1860.

NE = *Nouveaux Essais sur l'Entendement.* Cited by book, chapter, and section.

RB = *New Essays on Human Understanding.* Trans. by Peter Remnant and Jonathan Bennett. Cambridge: Cambridge University Press. 1982.

W = *G.W. Leibniz Philosophical Texts*, Trans and eds. Richard Francks and R.S. Woolhouse; with introduction and notes by R.S. Woolhouse. New York: Oxford University Press. 1998.

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[i] As Michel Fichant points out, the concept of primitive force has its historical origin in Aristotle's *De Anima* (Fichant 1998, p.211). The term "entelecheia" can be found in this work (De Anima 2.1.412b5).

[ii] Leibniz refers to the motions regarded as internal phenomena for a perceiver. For example, in a letter to Arnauld Leibniz states that a motion considered as a phenomenon is a immediate result or effect of another phenomenon in a mind (AG 82 = G2 92).

[iii] There is a dispute concerning whether simple substances literally "in" a body (Ross 1984, p.30-2; Adams 1994, p.248; Hoffman 1996, pp.113-5; Hartz 2007, p.151). However, I am taking a body to be an aggregate of simple substances that exist outside of a perceiver. Then it is natural to suppose that this aggregate is actually constituted by these simple substances and in that sense they are "in" this aggregate.

[iv] As well-known, that Leibniz emphasized that the quantity of motion is not conserved in nature. However, it is not the main point of his papers since many scientists (Christian Huygens, Christopher Wren etc.) already knew that the quantity of motion is not conserved. Considering this, Gregory Brown argues that Leibniz wrote the *Brevis Demonstratio* in order to show that mv^2 is the correct measure of force since it is conserved (Brown 1984, p.122 etc.). Also, Catharine Wilson states that mv^2 had a wholly different meaning for Leibniz (Wilson 1989, pp.142-5) since for Huygens it was a simple quantity without any metaphysical implications. It should be noted that Huygens already took the sum of mv^2 to be conserved in an elastic collision (Aiton 1985, p.131).

[v] Donald Rutherford emphasizes that Leibniz speaks of the innate or inborn force of bodies (G4 504-16; Rutherford 1995, p.242). Thus extension is not enough to constitute a substance.

[vi] George Mac Donald Ross argues that Leibniz equated the active aspect of a simple substance with its appetition, and the passive aspect with its perception (C 12-3; G2 281; Ross 1984, p.34). We may wonder if Leibniz's theory is that simple, but at least we can understand that if a simple substance has an active appetition or tendency to realize a certain event, that substance is regarded as active concerning the event.